Grade Four Mathematics Content Standards

By the end of grade four, students understand large numbers and addition, subtraction, multiplication, and division of whole numbers. They describe and compare simple fractions and decimals. They understand the properties of, and the relationships between, plane geometric figures. They collect, represent, and analyze data to answer questions.

Note: The sample problems illustrate the standards and are written to help clarify them. Some problems are written in a form that can be used directly with students; others will need to be modified, particularly in the primary grades, before they are used with students.

The symbol ⬬ identifies the key standards for grade four.

Number Sense

1.0 Students understand the place value of whole numbers and decimals to two decimal places and how whole numbers and decimals relate to simple fractions. Students use the concepts of negative numbers:

1.1 Read and write whole numbers in the millions.

Which of these is the number 5,005,014?
(CST released test question, 2004)

1. Five million, five hundred, fourteen
2. Five million, five thousand, fourteen
3. Five thousand, five hundred, fourteen
4. Five billion, five million, fourteen

1.2 Order and compare whole numbers and decimals to two decimal places.

Which is bigger: 3.1 or 3.09?

1.3 Round whole numbers through the millions to the nearest ten, hundred, thousand, ten thousand, or hundred thousand.

Two hundred twenty-four students attend Green Street School. Round this number to the nearest hundred.

Lunch was served to 3,778 students. Round this number to the nearest thousand.

Each year it is estimated that 42,225 Canadian geese migrate south to warmer climates. Round this number to the nearest ten thousand.

1.4 Decide when a rounded solution is called for and explain why such a solution may be appropriate.

Norberto has ten dollars and he wants to buy some ballpoint pens, which cost $2.35; some notebooks, which cost $4.40; and a fancy eraser, which costs $1.45. He wants to make sure he has enough money to pay for all of them, so he rounds the cost of each item to the nearest dollar and adds them up: $2 + $4 + $1 = $7. He concludes that his ten dollars would be sufficient to buy all the items. Is he correct and, if so, why? If the estimate that he makes turns out to be $8 instead of $7, should he be concerned?

1.5 Explain different interpretations of fractions, for example, parts of a whole, parts of a set, and division of whole numbers by whole numbers; explain equivalence of fractions (see Standard 4.0).

1.6 Write tenths and hundredths in decimal and fraction notations and know the fraction and decimal equivalents for halves and fourths (e.g., $\frac{1}{2} = 0.5$ or 0.50; $\frac{7}{4} = 1\frac{3}{4} = 1.75$).

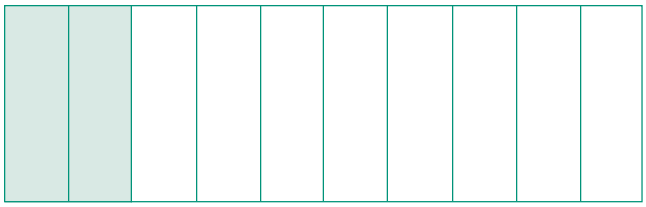
Which fraction means the same as 0.17?

(a) $\frac{17}{10}$ (b) $\frac{17}{100}$ (c) $\frac{17}{1000}$ (d) $\frac{17}{1}$

(CST released test question, 2004)

1.7 Write the fraction represented by a drawing of parts of a figure; represent a given fraction by using drawings; and relate a fraction to a simple decimal on a number line.

Which number represents the shaded part of the figure?
(Adapted from TIMSS gr. 3–4, M-5)



(a) 2.8
(b) 0.5
(c) 0.2
(d) 0.02

1.8 Use concepts of negative numbers (e.g., on a number line, in counting, in temperature, in "owing").

Yesterday's temperature was 5 degrees Celsius, but the temperature dropped 9 degrees Celsius overnight. What is today's temperature?

Determine if the following number sentences are true or false by identifying the relative positions of each number on a number line:

$-9 > -10$ $-31 < -29$

1.9 Identify on a number line the relative position of positive fractions, positive mixed numbers, and positive decimals to two decimal places.

Write a positive number for each letter on the number line shown below.

Which letter represents the number closest to 2.5?

Write the letter that represents where each number would go on the number line shown below:

$1\frac{1}{4}$ ___ 2.50 ___ $\frac{3}{4}$ ___

Determine if the following number sentences are true or false by identifying the relative positions of each number on a number line:

1. $\frac{1}{4} > 2.54$
2. $\frac{5}{2} < 2.6$
3. $\frac{12}{18} = \frac{2}{3}$ (Note the equivalence of fractions.)
4. $\frac{4}{5} < \frac{13}{15}$

2.0 Students extend their use and understanding of whole numbers to the addition and subtraction of simple decimals:

2.1 Estimate and compute the sum or difference of whole numbers and positive decimals to two places.

Solve 55.73 – 48.25 = ?

2.2 Round two-place decimals to one decimal or to the nearest whole number and judge the reasonableness of the rounded answer.

In her science class Li Ping weighs two samples of quartz and determines that the first has a weight of 3.44 grams and the second has a weight of 2.39 grams. Her teacher wants Li Ping to report the combined weight of the two samples to the nearest tenth of a gram, and to the nearest gram; however, the scale cannot measure weights over 5 grams. Li Ping decides to round the numbers first and then to add them.

1. Is 3.4 + 2.4 a reasonable estimate of the combined weights to the nearest tenth of a gram?
2. Is 3 + 2 a reasonable estimate of the combined weights to the nearest gram?

3.0 Students solve problems involving addition, subtraction, multiplication, and division of whole numbers and understand the relationships among the operations:

3.1 Demonstrate an understanding of, and the ability to use, standard algorithms for the addition and subtraction of multidigit numbers.

Solve these problems using the standard algorithms:

1. 619,581 – 23,183 = ?
2. 6,747 + 321,105 = ?

3.2 Demonstrate an understanding of, and the ability to use, standard algorithms for multiplying a multidigit number by a two-digit number and for dividing a multidigit number by a one-digit number; use relationships between them to simplify computations and to check results.

Singh and Sepideh work independently to solve the problem $783 \times 23 = ?$ They apply slightly different approaches, as shown below. Explain why both approaches are valid and give the same answer.

783	783	2,349	783
$\times 3$	$\times 20$	$+ 15{,}660$	$\times 23$
2,349	15,660	18,009	2,349
			$+ 15{,}660$
			18,009
Singh			Sepideh

3.3 Solve problems involving multiplication of multidigit numbers by two-digit numbers.

3.4 Solve problems involving division of multidigit numbers by one-digit numbers.

Solve each of the following problems and observe the different roles played by the number 37 in each situation:

1. Four children shared 37 dollars equally. How much did each get?
2. Four children shared 37 pennies as equally as possible. How many pennies did each get?
3. Cars need to be rented for 37 children going on a field trip. Each car can take 12 children in addition to the driver. How many cars must be rented?
4. There are 9 rows of seats in a theater. Each row has the same number of seats. If there is a total of 162 seats, how many seats are in each row? (CST released test question, 2004)

4.0 Students know how to factor small whole numbers:

4.1 Understand that many whole numbers break down in different ways (e.g., $12 = 4 \times 3 = 2 \times 6 = 2 \times 2 \times 3$).

In how many distinct ways can you write 60 as a product of two numbers?

4.2 Know that numbers such as 2, 3, 5, 7, and 11 do not have any factors except 1 and themselves and that such numbers are called prime numbers.

Circle all the prime numbers in these different representations of 24:

(a) 2×12 (c) 4×6 (e) $2 \times 3 \times 4$ (g) 1×24

(b) 3×8 (d) $2 \times 2 \times 6$ (f) $2 \times 2 \times 2 \times 3$

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Algebra and Functions

1.0 Students use and interpret variables, mathematical symbols, and properties to write and simplify expressions and sentences:

1.1 Use letters, boxes, or other symbols to stand for any number in simple expressions or equations (e.g., demonstrate an understanding and the use of the concept of a variable).

Tanya has read the first 78 pages of a 130-page book. Give the number sentence that can be used to find the number of pages Tanya must read to finish the book. (Adapted from TIMSS gr. 3–4, I-7)

1. $130 + 78 =$ ___
2. ___ $- 78 \square = 130$
3. $130 - 78 =$ ___
4. $130 -$ ___ $= 178$

1.2 Interpret and evaluate mathematical expressions that now use parentheses.

Evaluate the two expressions:

$(28 - 10) - 8 =$ ___ and $28 - (10 - 8) =$ ___.

Solve $5 \times (8 - 2) = ?$ (CST released test question, 2004)

1.3 Use parentheses to indicate which operation to perform first when writing expressions containing more than two terms and different operations.

What is the value of the expression below?

$(13 + 4) - (7 \times 2) + (31 - 17)$

(Adapted from CST released test question, 2004)

1.4 Use and interpret formulas (e.g., area = length $\times$ width or $A = lw$) to answer questions about quantities and their relationships.

Vik has a car that has a 16-gallon gas tank. When the tank is filled, he can drive 320 miles before running out of gas. How can Vik calculate his car's mileage in miles/gallon?

1.5 Understand that an equation such as $y = 3x + 5$ is a prescription for determining a second number when a first number is given.

2.0 **Students know how to manipulate equations:**

2.1 Know and understand that equals added to equals are equal.

The letters S and T stand for numbers. If $S - 100 = T - 100$, which statement is true? (CST released test question, 2004)

$S = T$ $S > T$ $S = T + 100$ $S > T + 100$

2.2 Know and understand that equals multiplied by equals are equal.

What number goes into the box to make this number sentence true? $(7 - 3) \times 5 = 4 \times$ [box] (CST released test question, 2004)

Measurement and Geometry

1.0 Students understand perimeter and area:

1.1 Measure the area of rectangular shapes by using appropriate units, such as square centimeter (cm^2), square meter (m^2), square kilometer (km^2), square inch ($in.^2$), square yard ($yd.^2$), or square mile ($mi.^2$).

1.2 Recognize that rectangles that have the same area can have different perimeters.

Draw a rectangle whose area is 120 and whose perimeter exceeds 50. Draw another rectangle with the same area whose perimeter exceeds 240.

1.3 Understand that rectangles that have the same perimeter can have different areas.

Is the area of a 45×55 rectangle (in cm^2) smaller or bigger than that of a square with the same perimeter?

Draw a rectangle whose perimeter is 40 and whose area is less than 20.

1.4 Understand and use formulas to solve problems involving perimeters and areas of rectangles and squares. Use those formulas to find the areas of more complex figures by dividing the figures into basic shapes.

The length of a rectangle is 6 cm, and its perimeter is 16 cm. What is the area of the rectangle in square centimeters? (TIMSS gr. 7–8, K-5)

2.0 **Students use two-dimensional coordinate grids to represent points and graph lines and simple figures:**

2.1 Draw the points corresponding to linear relationships on graph paper (e.g., draw 10 points on the graph of the equation $y = 3x$ and connect them by using a straight line).

1. Draw ten points on the graph of the equation $x = 4$.
2. Draw ten points on the graph of the equation $y = 71$.
3. Draw ten points on the graph of the equation $y = 2x + 4$.

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2.2 Understand that the length of a horizontal line segment equals the difference of the *x*-coordinates.

What is the length of the line segment joining the points (6, –4) and (21, –4)?

2.3 Understand that the length of a vertical line segment equals the difference of the *y*-coordinates.

What is the length of the line segment joining the points (121, 3) to (121, 17)?

3.0 **Students demonstrate an understanding of plane and solid geometric objects and use this knowledge to show relationships and solve problems:**

3.1 Identify lines that are parallel and perpendicular.

(Teachers are advised to introduce the terms *intersecting lines* and *nonintersecting lines* when dealing with this standard.)

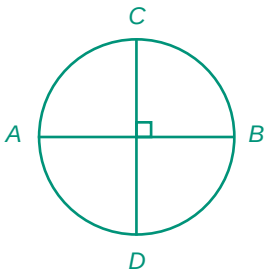
3.2 Identify the radius and diameter of a circle.

3.3 Identify congruent figures.

3.4 Identify figures that have bilateral and rotational symmetry.

Craig folded a piece of paper in half and cut out a shape along the folded edge. Draw a picture to show what the cutout shape will look like when it is opened up and flattened out.
(Adapted from TIMSS gr. 3–4, T-5)

Let *AB*, *CD* be perpendicular diameters of a circle, as shown. If we reflect across the line segment *CD*, what happens to *A* and what happens to *B* under this reflection?



3.5 Know the definitions of a right angle, an acute angle, and an obtuse angle. Understand that 90°, 180°, 270°, and 360° are associated, respectively, with $\frac{1}{4}$, $\frac{1}{2}$, $\frac{3}{4}$, and full turns.

3.6 Visualize, describe, and make models of geometric solids (e.g., prisms, pyramids) in terms of the number and shape of faces, edges, and vertices; interpret two-dimensional representations of three-dimensional objects; and draw patterns (of faces) for a solid that, when cut and folded, will make a model of the solid.

3.7 Know the definitions of different triangles (e.g., equilateral, isosceles, scalene) and identify their attributes.

Name each of the following triangles:

1. No equal sides
2. Two equal sides
3. Three equal sides

3.8 Know the definition of different quadrilaterals (e.g., rhombus, square, rectangle, parallelogram, trapezoid).

Explain which of the following statements are true and why:

1. All squares are rectangles.
2. All rectangles are squares.
3. All parallelograms are rectangles.
4. All rhombi are parallelograms.
5. Some parallelograms are squares.

Statistics, Data Analysis, and Probability

1.0 Students organize, represent, and interpret numerical and categorical data and clearly communicate their findings:

The following table shows the ages of the girls and boys in a club. Complete the graph by using the information for ages 9 and 10 shown in the table. (Adapted from TIMSS gr. 3–4, S-1)

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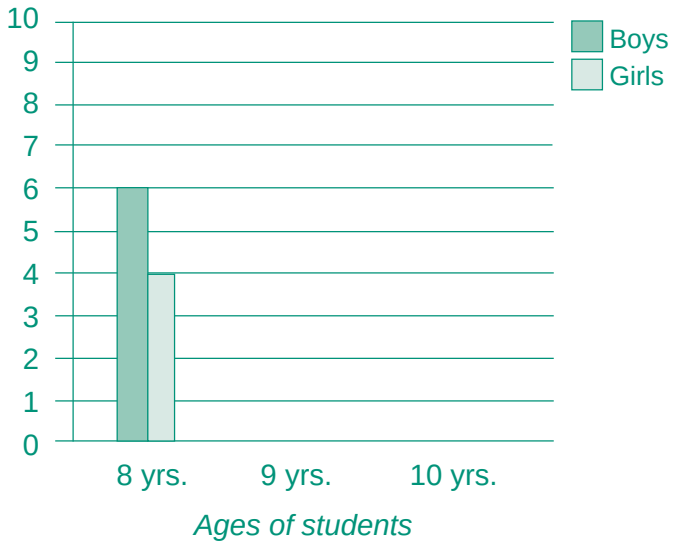
Ages	Number of Girls	Number of Boys
8	4	6
9	8	4
10	6	10



1.1 Formulate survey questions; systematically collect and represent data on a number line; and coordinate graphs, tables, and charts.

1.2 Identify the mode(s) for sets of categorical data and the mode(s), median, and any apparent outliers for numerical data sets.

1.3 Interpret one- and two-variable data graphs to answer questions about a situation.

2.0 Students make predictions for simple probability situations:

Nine identical chips numbered 1 through 9 are put in a jar. When a chip is drawn from the jar, what is the probability that it has an even number? (Adapted from TIMSS gr. 7–8, N-18)

2.1 Represent all possible outcomes for a simple probability situation in an organized way (e.g., tables, grids, tree diagrams).

2.2 Express outcomes of experimental probability situations verbally and numerically (e.g., 3 out of 4; $\frac{3}{4}$).

Royce has a bag with 8 red marbles, 4 blue marbles, 5 green marbles, and 9 yellow marbles all the same size. If he pulls out 1 marble without looking, which color is he most likely to choose? (CST released test question, 2004)

Mathematical Reasoning

1.0 Students make decisions about how to approach problems:

1.1 Analyze problems by identifying relationships, distinguishing relevant from irrelevant information, sequencing and prioritizing information, and observing patterns.

1.2 Determine when and how to break a problem into simpler parts.

2.0 Students use strategies, skills, and concepts in finding solutions:

2.1 Use estimation to verify the reasonableness of calculated results.

2.2 Apply strategies and results from simpler problems to more complex problems.

2.3 Use a variety of methods, such as words, numbers, symbols, charts, graphs, tables, diagrams, and models, to explain mathematical reasoning.

2.4 Express the solution clearly and logically by using the appropriate mathematical notation and terms and clear language; support solutions with evidence in both verbal and symbolic work.

2.5 Indicate the relative advantages of exact and approximate solutions to problems and give answers to a specified degree of accuracy.

2.6 Make precise calculations and check the validity of the results from the context of the problem.

3.0 Students move beyond a particular problem by generalizing to other situations:

3.1 Evaluate the reasonableness of the solution in the context of the original situation.

3.2 Note the method of deriving the solution and demonstrate a conceptual understanding of the derivation by solving similar problems.

3.3 Develop generalizations of the results obtained and apply them in other circumstances.

Grade Five Mathematics Content Standards

By the end of grade five, students increase their facility with the four basic arithmetic operations applied to fractions and decimals and learn to add and subtract positive and negative numbers. They know and use common measuring units to determine length and area and know and use formulas to determine the volume of simple geometric figures. Students know the concept of angle measurement and use a protractor and compass to solve problems. They use grids, tables, graphs, and charts to record and analyze data.

Note: The sample problems illustrate the standards and are written to help clarify them. Some problems are written in a form that can be used directly with students; others will need to be modified, particularly in the primary grades, before they are used with students.

The symbol ⬬ identifies the key standards for grade five.

Number Sense

1.0 Students compute with very large and very small numbers, positive integers, decimals, and fractions and understand the relationship between decimals, fractions, and percents. They understand the relative magnitudes of numbers:

1.1 Estimate, round, and manipulate very large (e.g., millions) and very small (e.g., thousandths) numbers.

⬬ **1.2** Interpret percents as a part of a hundred; find decimal and percent equivalents for common fractions and explain why they represent the same value; compute a given percent of a whole number.

What is 40% of 250? (CST released test question, 2004)

A test had 48 problems. Joe got 42 correct.

1. What percent were correct?
2. What percent were wrong?
3. If Moe got 93.75% correct, how many problems did he get correct?

1.3 Understand and compute positive integer powers of nonnegative integers; compute examples as repeated multiplication.

Which is bigger: 3^5 or 5^3?

⬬ **1.4** Determine the prime factors of all numbers through 50 and write the numbers as the product of their prime factors by using exponents to show multiples of a factor (e.g., $24 = 2 \times 2 \times 2 \times 3 = 2^3 \times 3$).

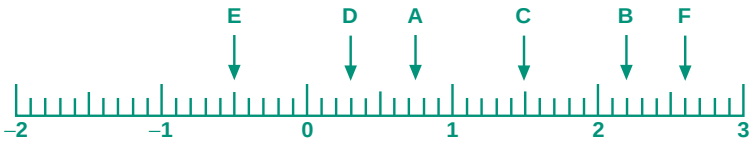
Find the prime factorization of 48 and use exponents where appropriate.

⬬ **1.5** Identify and represent on a number line decimals, fractions, mixed numbers, and positive and negative integers.

Next to each number, write the letter that represents the quantity on the number line.

2.2 _____ 0.3 _____ –0.5 _____

$2\frac{6}{10}$ _____ $\frac{75}{100}$ _____ 1.5 _____



Place the following numbers, in approximate positions, on the number line: $1\frac{3}{7}$, 1.43, $\frac{23}{14}$.

2.0 Students perform calculations and solve problems involving addition, subtraction, and simple multiplication and division of fractions and decimals:

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2.1 Add, subtract, multiply, and divide with decimals; add with negative integers; subtract positive integers from negative integers; and verify the reasonableness of the results.

Determine the following numbers:

1. $11 + (-23)$
2. $(-15) - 128$
3. $51 - 24.7$
4. 8.2×24.7
5. $68.13 \div 3$

2.2 Demonstrate proficiency with division, including division with positive decimals and long division with multidigit divisors.

Find the quotient: 6 divided by 0.025.

$15.12 \div 2.4 = ?$ (CST released test question, 2004)

2.3 Solve simple problems, including ones arising in concrete situations, involving the addition and subtraction of fractions and mixed numbers (like and unlike denominators of 20 or less), and express answers in the simplest form.

Suppose a galleon is a type of money worth 17 sickles. If Ludo borrows $2\frac{3}{17}$ galleons from Harry, then gives him back 12 sickles, how many galleons and sickles does Ludo still owe?

Sally is training to walk in a marathon. In her second week of training, she walked $5\frac{3}{4}$ miles on Tuesday, $5\frac{1}{16}$ miles on Thursday, and $16\frac{3}{8}$ miles on Sunday. How many miles altogether did Sally walk on those three days?

Jerry and Larry both ordered personal-sized pizzas for lunch. Jerry ate $\frac{3}{4}$ of his pizza, and Larry ate $\frac{2}{3}$ of his pizza. Who ate more pizza and how much more did he eat?

Given the following three pairs of fractions: $\frac{3}{8}$ and $\frac{1}{16}$, $5\frac{1}{4}$ and $1\frac{3}{4}$, 16 and $3\frac{1}{5}$, find for each pair its:

1. Sum
2. Difference

2.4 Understand the concept of multiplication and division of fractions.

$\frac{3}{4} \div \frac{3}{5} = \frac{?}{}$ (CST released test question, 2004)

2.5 Compute and perform simple multiplication and division of fractions and apply these procedures to solving problems.

Given the following three pairs of fractions: $\frac{3}{8}$ and $\frac{1}{16}$, $5\frac{1}{4}$ and $1\frac{3}{4}$, 16 and $3\frac{1}{5}$), find for each pair its:

1. Product
2. Quotient in simplest terms

Ericka has $3\frac{1}{2}$ yards of cloth to make shirts. Each shirt requires $\frac{7}{8}$ yard. How many shirts can she make? How much cloth will she have left over?

Algebra and Functions

1.0 Students use variables in simple expressions, compute the value of the expression for specific values of the variable, and plot and interpret the results:

1.1 Use information taken from a graph or equation to answer questions about a problem situation.

1.2 Use a letter to represent an unknown number; write and evaluate simple algebraic expressions in one variable by substitution.

If x is a number that satisfies $3x + 2 = 14$, can x be equal to 3?
If $N = 4$, what is the value of $6 \times N - 3$?
(CST released test question, 2004)

1.3 Know and use the distributive property in equations and expressions with variables.

What value for z makes this equation true?

$8 \times 37 = (8 \times 30) + (8 \times z)$ (CST released test question, 2004)

1.4 Identify and graph ordered pairs in the four quadrants of the coordinate plane.

Plot these points on a coordinate plane:
(1, 2), (–4, –3), (12, –1), (0, 4), (–4, 0)

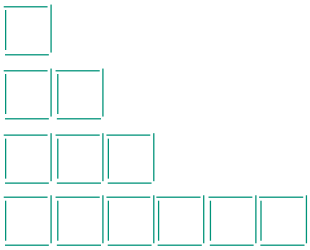
1.5 Solve problems involving linear functions with integer values; write the equation; and graph the resulting ordered pairs of integers on a grid.

Which equation could have been used to create this function table? (CST released test question, 2004)

x	y
–9	–5
–2	2
4	8
11	15

$y = \frac{x}{2}$ $y = 2x$ $y = x - 4$ $y = x + 4$

One can build rows of squares with toothpicks, as shown below for the case of 1, 2, 3, and 6 squares, respectively:



Explain why the following formula

$y = 3n + 1$

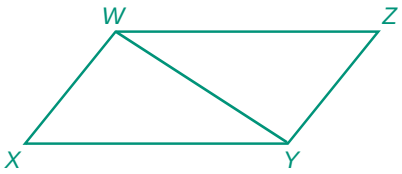
for the number of toothpicks y needed to form a row of n squares is correct. Graph this equation on a grid and remember that n takes on only whole number values 1, 2, 3, 4, . . .

Measurement and Geometry

1.0 Students understand and compute the volumes and areas of simple objects:

1.1 Derive and use the formula for the area of a triangle and of a parallelogram by comparing each with the formula for the area of a rectangle (i.e., two of the same triangles make a parallelogram with twice the area; a parallelogram is compared with a rectangle of the same area by pasting and cutting a right triangle on the parallelogram).

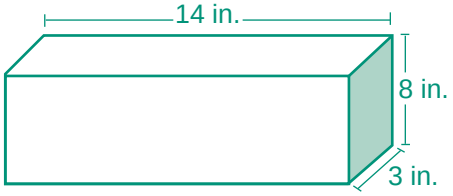
In the figure below, *WXYZ* is a parallelogram.



If the area of triangle *WXY* is 22 square inches, what is the area of *WXYZ*? (CST released test question, 2004)

1.2 Construct a cube and rectangular box from two-dimensional patterns and use these patterns to compute the surface area for these objects.

1.3 Understand the concept of volume and use the appropriate units in common measuring systems (i.e., cubic centimeter [cm^3], cubic meter [m^3], cubic inch [$in.^3$], cubic yard [$yd.^3$]) to compute the volume of rectangular solids.



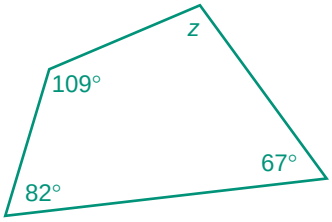
This rectangular prism has a length of 14 inches, a height of 8 inches, and a width of 3 inches. What is the volume? (CST released test question, 2004)

1.4 Differentiate between, and use appropriate units of measures for, two- and three-dimensional objects (i.e., find the perimeter, area, volume).

2.0 Students identify, describe, and classify the properties of, and the relationships between, plane and solid geometric figures:

2.1 Measure, identify, and draw angles, perpendicular and parallel lines, rectangles, and triangles by using appropriate tools (e.g., straightedge, ruler, compass, protractor, drawing software).

2.2 Know that the sum of the angles of any triangle is 180° and the sum of the angles of any quadrilateral is 360° and use this information to solve problems.



What is the measure of angle *z* in the figure above? (CST released test question, 2004)

2.3 Visualize and draw two-dimensional views of three-dimensional objects made from rectangular solids.

Statistics, Data Analysis, and Probability

1.0 Students display, analyze, compare, and interpret different data sets, including data sets of different sizes:

1.1 Know the concepts of mean, median, and mode; compute and compare simple examples to show that they may differ.

Compute the mean, median, and mode of the following collection of 27 numbers:

$\underbrace{1, 1, \ldots, 1}_{23}, 2, 3, 26, 135$

1.2 Organize and display single-variable data in appropriate graphs and representations (e.g., histogram, circle graphs) and explain which types of graphs are appropriate for various data sets.

1.3 Use fractions and percentages to compare data sets of different sizes.

1.4 Identify ordered pairs of data from a graph and interpret the meaning of the data in terms of the situation depicted by the graph.

1.5 Know how to write ordered pairs correctly; for example, (*x, y*).

Mathematical Reasoning

1.0 Students make decisions about how to approach problems:

1.1 Analyze problems by identifying relationships, distinguishing relevant from irrelevant information, sequencing and prioritizing information, and observing patterns.

1.2 Determine when and how to break a problem into simpler parts.

2.0 Students use strategies, skills, and concepts in finding solutions:

2.1 Use estimation to verify the reasonableness of calculated results.

2.2 Apply strategies and results from simpler problems to more complex problems.

2.3 Use a variety of methods, such as words, numbers, symbols, charts, graphs, tables, diagrams, and models, to explain mathematical reasoning.

2.4 Express the solution clearly and logically by using the appropriate mathematical notation and terms and clear language; support solutions with evidence in both verbal and symbolic work.

2.5 Indicate the relative advantages of exact and approximate solutions to problems and give answers to a specified degree of accuracy.

2.6 Make precise calculations and check the validity of the results from the context of the problem.

3.0 Students move beyond a particular problem by generalizing to other situations:

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3.1 Evaluate the reasonableness of the solution in the context of the original situation.

3.2 Note the method of deriving the solution and demonstrate a conceptual understanding of the derivation by solving similar problems.

3.3 Develop generalizations of the results obtained and apply them in other circumstances.

Grade Six Mathematics Content Standards

By the end of grade six, students have mastered the four arithmetic operations with whole numbers, positive fractions, positive decimals, and positive and negative integers; they accurately compute and solve problems. They apply their knowledge to statistics and probability. Students understand the concepts of mean, median, and mode of data sets and how to calculate the range. They analyze data and sampling processes for possible bias and misleading conclusions; they use addition and multiplication of fractions routinely to calculate the probabilities for compound events. Students conceptually understand and work with ratios and proportions; they compute percentages (e.g., tax, tips, interest). Students know about π and the formulas for the circumference and area of a circle. They use letters for numbers in formulas involving geometric shapes and in ratios to represent an unknown part of an expression. They solve one-step linear equations.

Number Sense

Note: The sample problems illustrate the standards and are written to help clarify them. Some problems are written in a form that can be used directly with students; others will need to be modified, particularly in the primary grades, before they are used with students.

The symbol ● identifies the key standards for grade six.

1.0 **Students compare and order positive and negative fractions, decimals, and mixed numbers. Students solve problems involving fractions, ratios, proportions, and percentages:**

1.1 Compare and order positive and negative fractions, decimals, and mixed numbers and place them on a number line.

Order the following numbers: $\frac{20}{21}$ $-\frac{4}{9}$ -4.4 $1\frac{1}{12}$ 1.1 $\frac{3}{7}$

If you were to place $-\frac{2}{3}$, -3, and $-\frac{7}{8}$ on a number line, which number would be closest to -1? Use a number line to explain your answer.

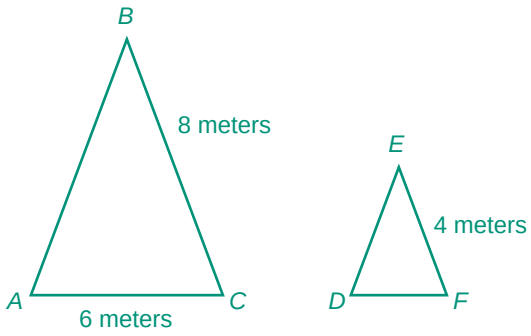
Place the following numbers on a number line:

0.3 $-\frac{3}{10}$ $2\frac{1}{2}$ $\frac{4}{5}$ $\frac{7}{8}$ -2

1.2 Interpret and use ratios in different contexts (e.g., batting averages, miles per hour) to show the relative sizes of two quantities, using appropriate notations (*a*/*b, a* to *b*, *a*:*b*).

1.3 Use proportions to solve problems (e.g., determine the value of *N* if $\frac{4}{7} = \frac{N}{21}$, find the length of a side of a polygon similar to a known polygon). Use cross-multiplication as a method for solving such problems, understanding it as the multiplication of both sides of an equation by a multiplicative inverse.

$\triangle ABC$ is similar to $\triangle DEF$. What is the length of $\overline{DF}$?
(CST released test question, 2004)



Ballpoint pens are sold in bundles of four. Lee bought 24 pens for $14.40. How much would 56 pens cost? Carefully explain your solution.

Find n if:

1. $\frac{49}{21} = \frac{14}{n}$

2. $\frac{28}{n} = \frac{36}{27}$

(This problem also applies to Algebra and Functions Standard 1.1.)

1.4 Calculate given percentages of quantities and solve problems involving discounts at sales, interest earned, and tips.

Ann paid $70.20 for a dress, and the amount includes an 8% sales tax. What is the cost of the dress before the tax?

2.0 Students calculate and solve problems involving addition, subtraction, multiplication, and division:

2.1 Solve problems involving addition, subtraction, multiplication, and division of positive fractions and explain why a particular operation was used for a given situation.

Your after-school program is on a hiking trip. You hike $\frac{3}{4}$ of a mile and stop to rest. Your friend hikes $\frac{4}{5}$ of a mile, then turns around and hikes back $\frac{1}{8}$ of a mile. Who is farther ahead on the trail? How much farther? Explain how you solved the problem.

At soccer practice the team has to run around a rectangular field that is $75\frac{1}{2}$ feet by $127\frac{3}{4}$ feet. The coach makes the team run around the field three times. How many total feet does a team member run? Explain how you solved this problem.

Mario wants to make half of his special no-bake cookie recipe. The recipe calls for $1\frac{3}{4}$ cups of white sugar, $\frac{1}{3}$ cup of margarine, $\frac{1}{2}$ cup of

peanut butter, and $3\frac{1}{4}$ cups of oats. How much of each ingredient will Mario need? Explain how you solved this problem.

Jim was on a hiking trail and after walking $\frac{3}{4}$ of a mile, he found that he was only $\frac{5}{8}$ of the way to the end of the trail. How long is the trail? Explain.

2.2 Explain the meaning of multiplication and division of positive fractions and perform the calculations (e.g., $\frac{5}{8} \div \frac{15}{16} = \frac{5}{8} \times \frac{16}{15} = \frac{2}{3}$).

Draw a picture that illustrates each of the following problems and its solution. Explain how your drawings illustrate the problems and the solutions.

1. $\frac{3}{4} \times \frac{1}{2}$
2. $\frac{3}{4} \div \frac{1}{2}$
3. $2 \times \frac{1}{4}$

2.3 Solve addition, subtraction, multiplication, and division problems, including those arising in concrete situations, that use positive and negative integers and combinations of these operations.

Two friends start out on a daylong hike. They start at an elevation of 526 feet. The morning hike takes them to an altitude 300 feet higher than where they started. In the afternoon the friends descend 117 feet and stop to rest. Then they continue downward and descend another 366 feet. Describe the change in altitude.

Simplify to make the calculation as simple as possible:

1. $-19 + 37 + 19$
2. $(-16)(-28) + (-16)27$
3. $(-8)(-4)(19)(6 + (-6))$

2.4 Determine the least common multiple and the greatest common divisor of whole numbers; use them to solve problems with fractions (e.g., to find a common denominator to add two fractions or to find the reduced form for a fraction).

$\frac{3}{8} + \frac{1}{2} = ?$ (CST released test question, 2004)

Algebra and Functions

1.0 Students write verbal expressions and sentences as algebraic expressions and equations; they evaluate algebraic expressions, solve simple linear equations, and graph and interpret their results:

Grade Six

1.1 Write and solve one-step linear equations in one variable.

What value of k makes the following equation true?

$k \div 3 = 36$ (CST released test question, 2004)

$y - 2 = 10$. What is y?

$6y = 12$. What is y?

If a number y satisfies $y + 17 = 10$, what is y? If a number x satisfies $3x = 25$, what is x?

1.2 Write and evaluate an algebraic expression for a given situation, using up to three variables.

A telephone company charges $0.05 per minute for local calls and $0.12 per minute for long-distance calls. Which expression gives the total cost in dollars for x minutes of local calls and y minutes of long-distance calls? (CST released test question, 2004)

(a) $0.05x + 0.12y$
(b) $0.05x - 0.12y$
(c) $0.17(x + y)$
(d) $0.17xy$

1.3 Apply algebraic order of operations and the commutative, associative, and distributive properties to evaluate expressions; and justify each step in the process.

Simplify:

1. $(4^3 + 7) - (5 - 8)^3$
2. $11[5(7^2) - 3^2 - 12(20 + 5.4 + 2)]$
3. $-3 \cdot (3^2 + 3) \div 3^2$

1.4 Solve problems manually by using the correct order of operations or by using a scientific calculator.

2.0 Students analyze and use tables, graphs, and rules to solve problems involving rates and proportions:

2.1 Convert one unit of measurement to another (e.g., from feet to miles, from centimeters to inches).

Suppose that one British pound is worth $1.50. In London a magazine costs 3 pounds. In San Francisco the same magazine costs $4.25. In which city is the magazine cheaper?

When temperature is measured in both Celsius (C) and Fahrenheit (F), it is known that they are related by the following formula:

$9 \times C = (F - 32) \times 5$. What is 50 degrees Fahrenheit in Celsius?

(Note the explicit use of parentheses.)

How many inches are in $2\frac{1}{2}$ feet? (CST released test question, 2004)

2.2 Demonstrate an understanding that *rate* is a measure of one quantity per unit value of another quantity.

Grade Six

Joe can type 9 words in 8 seconds. At this rate, how many words can he type in 2 minutes?

2.3 Solve problems involving rates, average speed, distance, and time.

Marcus took a train from San Francisco to San Jose, a distance of 54 miles. The train took 45 minutes for the trip. What was the average speed of the train?

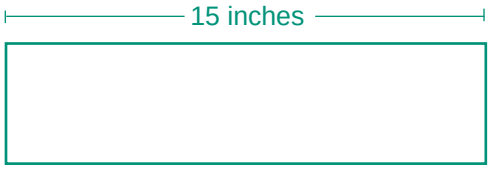
3.0 Students investigate geometric patterns and describe them algebraically:

3.1 Use variables in expressions describing geometric quantities (e.g., $P = 2w + 2l$, $A = \frac{1}{2}bh$, $C = \pi d$—the formulas for the perimeter of a rectangle, the area of a triangle, and the circumference of a circle, respectively).

A rectangle has width w. Its length is one more than 3 times its width. Find the perimeter of the rectangle. (Your answer will be expressed in terms of w.)

3.2 Express in symbolic form simple relationships arising from geometry.

The rectangle shown below has length 15 inches and perimeter P inches.



Which equation could be used to find the width of the rectangle?

$P = 15 + \frac{w}{2}$ $P = 15 - w$ $P = 30 + 2w$ $P = 30 - 2w$

(CST released test question, 2004)

Measurement and Geometry

1.0 Students deepen their understanding of the measurement of plane and solid shapes and use this understanding to solve problems:

1.1 Understand the concept of a constant such as π; know the formulas for the circumference and area of a circle.

Grade Six

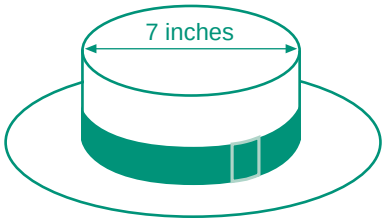
Which equation could be used to find the area in square inches of a circle with a radius of 8 inches? (CST released test question, 2004)

(a) $A = 4 \times \pi$ (b) $A = \pi \times 4^2$ (c) $A = 8 \times \pi$ (d) $A = \pi \times 8^2$

1.2 Know common estimates of π (3.14; $\frac{22}{7}$) and use these values to estimate and calculate the circumference and the area of circles; compare with actual measurements.

What is the circumference of a circle with a radius of 5? (Answer: 10π or approximately 31.4)

The top part of this hat is shaped like a cylinder with a diameter of 7 inches.

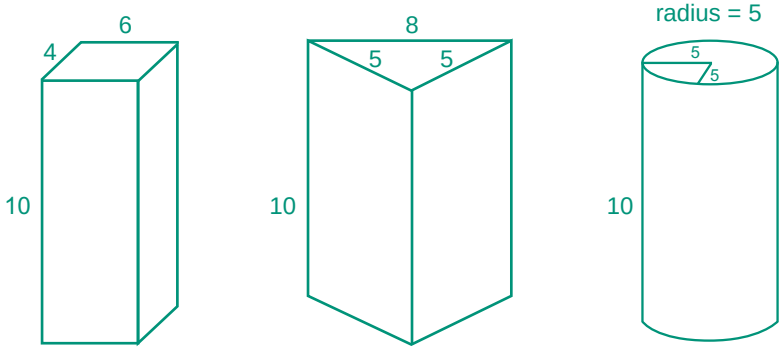


Which measure is *closest* to the length of the band that goes around the outside of the hat? (CST released test question, 2004)

(a) 10.1 inches (b) 11.0 inches (c) 22.0 inches (d) 38.5 inches

1.3 Know and use the formulas for the volume of triangular prisms and cylinders (area of base × height); compare these formulas and explain the similarity between them and the formula for the volume of a rectangular solid.

Find the volumes (dimensions are in cm).

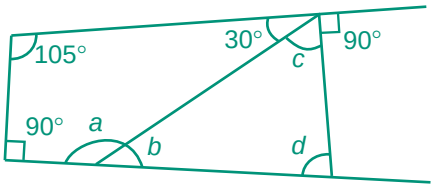


2.0 Students identify and describe the properties of two-dimensional figures:

2.1 Identify angles as vertical, adjacent, complementary, or supplementary and provide descriptions of these terms.

2.2 Use the properties of complementary and supplementary angles and the sum of the angles of a triangle to solve problems involving an unknown angle.

Find the missing angles *a, b, c,* and *d.*



2.3 Draw quadrilaterals and triangles from given information about them (e.g., a quadrilateral having equal sides but no right angles, a right isosceles triangle).

Statistics, Data Analysis, and Probability

1.0 Students compute and analyze statistical measurements for data sets:

1.1 Compute the range, mean, median, and mode of data sets.

1.2 Understand how additional data added to data sets may affect these computations.

1.3 Understand how the inclusion or exclusion of outliers affects these computations.

1.4 Know why a specific measure of central tendency (mean, median) provides the most useful information in a given context.

2.0 Students use data samples of a population and describe the characteristics and limitations of the samples:

2.1 Compare different samples of a population with the data from the entire population and identify a situation in which it makes sense to use a sample.

2.2 Identify different ways of selecting a sample (e.g., convenience sampling, responses to a survey, random sampling) and which method makes a sample more representative for a population.

2.3 Analyze data displays and explain why the way in which the question was asked might have influenced the results obtained and why the way in which the results were displayed might have influenced the conclusions reached.

2.4 Identify data that represent sampling errors and explain why the sample (and the display) might be biased.

Grade Six

2.5 Identify claims based on statistical data and, in simple cases, evaluate the validity of the claims.

Calvin has been identified as the best runner in your school because he won the 50-yard dash at the all-schools track meet. Use the records of the track team in the table shown below to decide if Calvin is the best runner in the school. Explain your decision, using the data in the table.

Runner	*Race 1*	*Race 2*	*Race 3*	*Race 4*
Brian	27.3	27.6	30.1	26.2
Maria	26.5	26.3	26.0	27.1
Calvin	30.2	28.1	29.4	25.0
Alice	28.2	29.0	32.0	27.4
Fred	32.1	32.5	29.0	30.0
José	26.2	26.0	25.8	25.5

Soraya has been assigned to do a survey for the student council. However, she forgets to do this task until the morning of the meeting, so she asks three of her best friends what kind of music they would like for a noon-time dance. Their opinions are what Soraya will report to student council.

Do you think Soraya's report is an accurate reflection of the kind of music that students want played for the noon-time dance? Explain your answer.

3.0 Students determine theoretical and experimental probabilities and use these to make predictions about events:

3.1 Represent all possible outcomes for compound events in an organized way (e.g., tables, grids, tree diagrams) and express the theoretical probability of each outcome.

3.2 Use data to estimate the probability of future events (e.g., batting averages or number of accidents per mile driven).

3.3 Represent probabilities as ratios, proportions, decimals between 0 and 1, and percentages between 0 and 100 and verify that the probabilities computed are reasonable; know that if *P* is the probability of an event, 1-*P* is the probability of an event not occurring.

3.4 Understand that the probability of either of two disjoint events occurring is the sum of the two individual probabilities and that the probability of one event following another, in independent trials, is the product of the two probabilities.

3.5 Understand the difference between independent and dependent events.

Mathematical Reasoning

1.0 Students make decisions about how to approach problems:

1.1 Analyze problems by identifying relationships, distinguishing relevant from irrelevant information, identifying missing information, sequencing and prioritizing information, and observing patterns.

1.2 Formulate and justify mathematical conjectures based on a general description of the mathematical question or problem posed.

1.3 Determine when and how to break a problem into simpler parts.

2.0 Students use strategies, skills, and concepts in finding solutions:

2.1 Use estimation to verify the reasonableness of calculated results.

2.2 Apply strategies and results from simpler problems to more complex problems.

2.3 Estimate unknown quantities graphically and solve for them by using logical reasoning and arithmetic and algebraic techniques.

2.4 Use a variety of methods, such as words, numbers, symbols, charts, graphs, tables, diagrams, and models, to explain mathematical reasoning.

2.5 Express the solution clearly and logically by using the appropriate mathematical notation and terms and clear language; support solutions with evidence in both verbal and symbolic work.

2.6 Indicate the relative advantages of exact and approximate solutions to problems and give answers to a specified degree of accuracy.

2.7 Make precise calculations and check the validity of the results from the context of the problem.

3.0 Students move beyond a particular problem by generalizing to other situations:

3.1 Evaluate the reasonableness of the solution in the context of the original situation.

3.2 Note the method of deriving the solution and demonstrate a conceptual understanding of the derivation by solving similar problems.

3.3 Develop generalizations of the results obtained and the strategies used and apply them in new problem situations.

Grade Seven Mathematics Content Standards

By the end of grade seven, students are adept at manipulating numbers and equations and understand the general principles at work. Students understand and use factoring of numerators and denominators and properties of exponents. They know the Pythagorean theorem and solve problems in which they compute the length of an unknown side. Students know how to compute the surface area and volume of basic three-dimensional objects and understand how area and volume change with a change in scale. Students make conversions between different units of measurement. They know and use different representations of fractional numbers (fractions, decimals, and percents) and are proficient at changing from one to another. They increase their facility with ratio and proportion, compute percents of increase and decrease, and compute simple and compound interest. They graph linear functions and understand the idea of slope and its relation to ratio.

Note: The sample problems illustrate the standards and are written to help clarify them. Some problems are written in a form that can be used directly with students; others will need to be modified, particularly in the primary grades, before they are used with students.

The symbol ⬬ identifies the key standards for grade seven.

Number Sense

1.0 Students know the properties of, and compute with, rational numbers expressed in a variety of forms:

1.1 Read, write, and compare rational numbers in scientific notation (positive and negative powers of 10), compare rational numbers in general.

Put the following numbers on the number line:

$-3.14 \quad -3.3 \quad -3\frac{1}{3} \quad -3.1 \quad -\frac{27}{8}$

Arrange the following numbers in increasing order:

$1.86 \times 10^5 \quad 185{,}766 \quad 1.004 \times 10^6 \quad 2.1 \times 10^5 \quad 205{,}666$

Arrange the following numbers in increasing order:

$-3.14 \times 10^{-2} \qquad 3.14 \times 10^2 \qquad -3.14 \times 10^2 \qquad 3.14 \times 10^{-2}$

⬬ 1.2 Add, subtract, multiply, and divide rational numbers (integers, fractions, and terminating decimals) and take positive rational numbers to whole-number powers.

1. $\frac{1}{4} \times 0.33$

2. $\dfrac{2\frac{1}{7}}{\frac{2}{3}} - \dfrac{\frac{3}{7}}{\left(\frac{2}{3}\right)^2}$

3. Evaluate:

$\frac{12}{7} \times \frac{6}{5} \times \frac{7}{8} =$

$3\frac{1}{5} \div (-5) =$

$(0.2)^5 \times \left(\frac{3}{2}\right)^4 =$

$\frac{1}{2}(58.3 - 11.29) =$

1.3 Convert fractions to decimals and percents and use these representations in estimations, computations, and applications.

Change to decimals:

$\frac{7}{8}$ $\frac{7}{11}$

1.4 Differentiate between rational and irrational numbers.

Which is an irrational number? (CST released test question, 2004)

(a) $\sqrt{5}$ (b) $\sqrt{9}$ (c) -1 (d) $-\frac{2}{3}$

1.5 Know that every rational number is either a terminating or a repeating decimal and be able to convert terminating decimals into reduced fractions.

Change to fractions:

0.25 $0.\overline{27}$

Find the period of the repeating part of $\frac{41}{13}$.

1.6 Calculate the percentage of increases and decreases of a quantity.

A sweater originally cost $37.50. Last week Moesha bought it at 20% off.

How much was deducted from the original price? (CST released test question, 2004)

(a) $7.50 (b) $17.50 (c) $20.00 (d) $30.00

Chapter 2
Mathematics
Content
Standards

1.7 Solve problems that involve discounts, markups, commissions, and profit and compute simple and compound interest.

Heather deposits $800 in an account that earns a flat rate of 10% (simple) interest. Jim deposits $800 in an account that earns 10% interest compounded yearly. Who will have more money at the end of one year? Two years? Three years? Who will have more money over the long run? Explain why.

Grade Seven

Jason bought a jacket on sale for 50% off the original price and another 25% off the discounted price. If the jacket originally cost $88, what was the final sale price that Jason paid for the jacket? (CST released test question, 2004)

2.0 Students use exponents, powers, and roots and use exponents in working with fractions:

2.1 Understand negative whole-number exponents. Multiply and divide expressions involving exponents with a common base.

Simplify:

1. $\dfrac{\left(\frac{2}{7}\right)^{5} \times \left(\frac{2}{7}\right)^{-11}}{\left(\frac{2}{7}\right)^{-3}}$

2. $\left(\frac{2}{3}\right)^{-3} \times \frac{2}{9}$

2.2 Add and subtract fractions by using factoring to find common denominators.

Make use of prime factors to compute:

1. $\frac{2}{28} + \frac{1}{49}$

2. $\frac{-5}{63} + \left(\frac{-7}{99}\right)$

2.3 Multiply, divide, and simplify rational numbers by using exponent rules.

Simplify:

1. $\dfrac{\left(-\frac{2}{3}\right)^{-3}}{2\frac{1}{4}} + \left(\frac{3}{-2}\right)^{2}\left(4 - 3\frac{1}{3}\right)$

2. $\dfrac{\left(\frac{2}{5} \times 2\frac{1}{3}\right)^{4}}{\left(\frac{2}{5}\right)\left(-2\frac{1}{3}\right)^{3}}$

3. $\frac{3^{-2}}{2^{-3}}$

4. $\frac{2x^3}{2^3x^{-1}}$

5. $\frac{4^2 \cdot 3^5 \cdot 2^4}{4^3 \cdot 3^5 \cdot 2^2}$ (CST released test question, 2004)

2.4 Use the inverse relationship between raising to a power and extracting the root of a perfect square integer; for an integer that is not square, determine without a calculator the two integers between which its square root lies and explain why.

Find the length of one side of a square that has an area of 81.

2.5 Understand the meaning of the absolute value of a number; interpret the absolute value as the distance of the number from zero on a number line; and determine the absolute value of real numbers.

Is it always true that for any numbers a and b, $a - |b| \leq a + b$?
$|9 - 5| - |6 - 8| = ?$ (CST released test question, 2004)

Algebra and Functions

1.0 Students express quantitative relationships by using algebraic terminology, expressions, equations, inequalities, and graphs:

1.1 Use variables and appropriate operations to write an expression, an equation, an inequality, or a system of equations or inequalities that represents a verbal description (e.g., three less than a number, half as large as area A).

Write the following verbal statements as algebraic expressions:

1. The square of a is increased by the sum of twice a and 3.
2. The product of $\frac{1}{2}$ of a and 3 is decreased by the quotient of a divided by (–4).

1.2 Use the correct order of operations to evaluate algebraic expressions such as $3(2x + 5)^2$.

Given $x = (-2)$ and $y = 5$ evaluate:

1. $x^2 + 2x - 3$
2. $\frac{y\ (xy\ -\ 7)}{10}$

1.3 Simplify numerical expressions by applying properties of rational numbers (e.g., identity, inverse, distributive, associative, commutative) and justify the process used.

Name the property illustrated by each of the following:

1. $y + -y = 0$
2. $x\,(y + z) = xy + xz$
3. $x\,(y + z) = (y + z)x$
4. $x + y = y + x$
5. $y\left(\frac{1}{y}\right) = 1$

1.4 Use algebraic terminology (e.g., variable, equation, term, coefficient, inequality, expression, constant) correctly.

1.5 Represent quantitative relationships graphically and interpret the meaning of a specific part of a graph in the situation represented by the graph.

2.0 Students interpret and evaluate expressions involving integer powers and simple roots:

2.1 Interpret positive whole-number powers as repeated multiplication and negative whole-number powers as repeated division or multiplication by the multiplicative inverse. Simplify and evaluate expressions that include exponents.

2.2 Multiply and divide monomials; extend the process of taking powers and extracting roots to monomials when the latter results in a monomial with an integer exponent.

3.0 Students graph and interpret linear and some nonlinear functions:

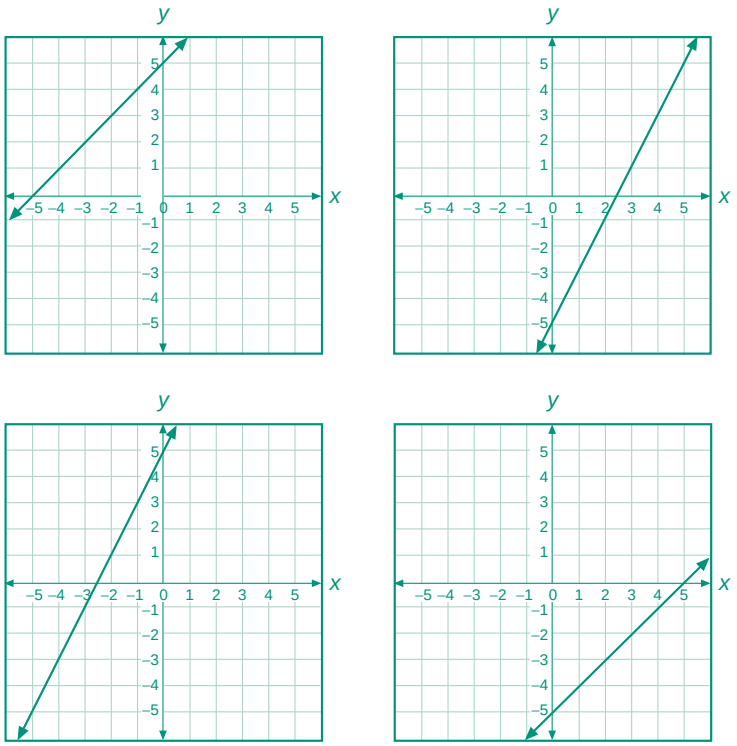
3.1 Graph functions of the form $y = nx^2$ and $y = nx^3$ and use in solving problems.

3.2 Plot the values from the volumes of three-dimensional shapes for various values of the edge lengths (e.g., cubes with varying edge lengths or a triangle prism with a fixed height and an equilateral triangle base of varying lengths).

3.3 Graph linear functions, noting that the vertical change (change in *y*-value) per unit of horizontal change (change in *x*-value) is always the same and know that the ratio (“rise over run”) is called the slope of a graph.

A function of x has value 7 when $x = 1$; it has value 15.5 when $x = 3.5$; and it has value 20 when $x = 5$. Is this a linear function?

Which best represents the graph of $y = 2x - 5$? (CST released test question, 2004)



3.4 Plot the values of quantities whose ratios are always the same (e.g., cost to the number of an item, feet to inches, circumference to diameter of a circle). Fit a line to the plot and understand that the slope of the line equals the ratio of the quantities.

4.0 Students solve simple linear equations and inequalities over the rational numbers:

4.1 Solve two-step linear equations and inequalities in one variable over the rational numbers, interpret the solution or solutions in the context from which they arose, and verify the reasonableness of the results.

Solve for x if $3x - 12 = 3{,}821$. If x stands for the number of books in a bookstore, can it satisfy this equation?

What is the solution set to the inequality $6z + 5 > 35$? (CST released test question, 2004)

$\{z : z < 5\}$ $\{z : z < 24\}$ $\{z : z > 5\}$ $\{z : z > 24\}$

4.2 Solve multistep problems involving rate, average speed, distance, and time or a direct variation.

A train can travel at either of two speeds between two towns that are 72 miles apart. The higher speed is 25% faster than the lower speed and reduces the travel time by 30 minutes. What are the two speeds in miles per hour?

A duck flew at 18 miles per hour for 3 hours, then at 15 miles per hour for 2 hours. How far did the duck fly in all? (CST released test question, 2004)

Juanita earns $36 for 3 hours of work. At that rate how long would she have to work to earn $720? (CST released test question, 2004)

Grade Seven

Measurement and Geometry

1.0 Students choose appropriate units of measure and use ratios to convert within and between measurement systems to solve problems:

1.1 Compare weights, capacities, geometric measures, times, and temperatures within and between measurement systems (e.g., miles per hour and feet per second, cubic inches to cubic centimeters).

Convert the following:

1. 80 miles/hr. = ? ft./sec.
2. 20 oz./min. = ? qts./day

1.2 Construct and read drawings and models made to scale.

1.3 Use measures expressed as rates (e.g., speed, density) and measures expressed as products (e.g., person-days) to solve problems; check the units of the solutions; and use dimensional analysis to check the reasonableness of the answer.

The chart shown below describes the speed of four printers.

Printer	Description
Roboprint	Prints 2 pages per second
Voltronn	Prints 1 page every 2 seconds
Vantek Plus	Prints 160 pages in 2 minutes
DLS Pro	Prints 100 pages per minute

Which printer is the fastest? (CST released test question, 2004)

2.0 Students compute the perimeter, area, and volume of common geometric objects and use the results to find measures of less common objects. They know how perimeter, area, and volume are affected by changes of scale:

2.1 Use formulas routinely for finding the perimeter and area of basic two-dimensional figures and the surface area and volume of basic three-dimensional figures, including rectangles, parallelograms, trapezoids, squares, triangles, circles, prisms, and cylinders.

2.2 Estimate and compute the area of more complex or irregular two- and three-dimensional figures by breaking the figures down into more basic geometric objects.

2.3 Compute the length of the perimeter, the surface area of the faces, and the volume of a three-dimensional object built from rectangular solids. Understand that when the lengths of all dimensions are multiplied by a scale factor, the surface area is multiplied by the square of the scale factor and the volume is multiplied by the cube of the scale factor.

2.4 Relate the changes in measurement with a change of scale to the units used (e.g., square inches, cubic feet) and to conversions between units (1 square foot = 144 square inches or [1 ft.^{2}] = [144 in.^{2}]; 1 cubic inch is approximately 16.38 cubic centimeters or [1 in.^{3}] = [16.38 cm^3]).

3.0 Students know the Pythagorean theorem and deepen their understanding of plane and solid geometric shapes by constructing figures that meet given conditions and by identifying attributes of figures:

3.1 Identify and construct basic elements of geometric figures (e.g., altitudes, midpoints, diagonals, angle bisectors, and perpendicular bisectors; central angles, radii, diameters, and chords of circles) by using a compass and straightedge.

3.2 Understand and use coordinate graphs to plot simple figures, determine lengths and areas related to them, and determine their image under translations and reflections.

3.3 Know and understand the Pythagorean theorem and its converse and use it to find the length of the missing side of a right triangle and the lengths of other line segments and, in some situations, empirically verify the Pythagorean theorem by direct measurement.

What is the side length of an isosceles right triangle with hypotenuse $\sqrt{72}$?

A right triangle has sides of lengths *a, b,* and *c*; *c* is the length of the hypotenuse. How would the areas of the three equilateral triangles with sides of lengths *a, b, c,* respectively, be related to each other?

3.4 Demonstrate an understanding of conditions that indicate two geometrical figures are congruent and what congruence means about the relationships between the sides and angles of the two figures.

3.5 Construct two-dimensional patterns for three-dimensional models, such as cylinders, prisms, and cones.

3.6 Identify elements of three-dimensional geometric objects (e.g., diagonals of rectangular solids) and describe how two or more objects are related in space (e.g., skew lines, the possible ways three planes might intersect).

True or false? If true, give an example. If false, explain why.

Two planes in three-dimensional space can:

1. Intersect in a line.
2. Intersect in a single point.
3. Have no intersection at all.

Statistics, Data Analysis, and Probability

1.0 Students collect, organize, and represent data sets that have one or more variables and identify relationships among variables within a data set by hand and through the use of an electronic spreadsheet software program:

1.1 Know various forms of display for data sets, including a stem-and-leaf plot or box-and-whisker plot; use the forms to display a single set of data or to compare two sets of data.

1.2 Represent two numerical variables on a scatterplot and informally describe how the data points are distributed and any apparent relationship that exists between the two variables (e.g., between time spent on homework and grade level).

1.3 Understand the meaning of, and be able to compute, the minimum, the lower quartile, the median, the upper quartile, and the maximum of a data set.

Here is a set of data for an exam in a mathematics class:

Minimum	45
Lower quartile score	51
Median	64
Upper quartile score	72
Maximum	92

1. Suppose there are 15 students in the class. Give a range of scores that would satisfy all the data shown above.
2. Suppose 7 students have scores ranging from 64 to 72. How many students might there be in the class? Explain.

Mathematical Reasoning

1.0 Students make decisions about how to approach problems:

1.1 Analyze problems by identifying relationships, distinguishing relevant from irrelevant information, identifying missing information, sequencing and prioritizing information, and observing patterns.

1.2 Formulate and justify mathematical conjectures based on a general description of the mathematical question or problem posed.

1.3 Determine when and how to break a problem into simpler parts.

2.0 Students use strategies, skills, and concepts in finding solutions:

2.1 Use estimation to verify the reasonableness of calculated results.

2.2 Apply strategies and results from simpler problems to more complex problems.

2.3 Estimate unknown quantities graphically and solve for them by using logical reasoning and arithmetic and algebraic techniques.

2.4 Make and test conjectures by using both inductive and deductive reasoning.

2.5 Use a variety of methods, such as words, numbers, symbols, charts, graphs, tables, diagrams, and models, to explain mathematical reasoning.

2.6 Express the solution clearly and logically by using the appropriate mathematical notation and terms and clear language; support solutions with evidence in both verbal and symbolic work.

2.7 Indicate the relative advantages of exact and approximate solutions to problems and give answers to a specified degree of accuracy.

2.8 Make precise calculations and check the validity of the results from the context of the problem.

3.0 Students determine a solution is complete and move beyond a particular problem by generalizing to other situations:

3.1 Evaluate the reasonableness of the solution in the context of the original situation.

3.2 Note the method of deriving the solution and demonstrate a conceptual understanding of the derivation by solving similar problems.

3.3 Develop generalizations of the results obtained and the strategies used and apply them to new problem situations.